Factorization of $n = 87463$ with the Quadratic Sieve

To find a factor base consider the values of $\left(\frac{n}{p}\right)$:

p	2	3	5	7	11	13	17	19	23	29	31	37
$\left(\frac{n}{p}\right)$	1	1	-1	-1	-1	1	1	1	-1	1	-1	-1

We thus select the factor base $2, 3, 13, 17, 19, 29$.

Solutions for $x^2 \equiv n \pmod{p}$ are:

p	2	3	13	17	19	29
x	1	1, 2	5, 8	7, 10	5, 14	12, 17

We now start sieving, using a sieving interval of length $2 \cdot 30$ around $\lfloor\sqrt{n}\rfloor = 295$.

For the values of x for which $x^2 - n$ splits completely, the exponent vector modulo 2 is:

x	-1	2	3	13	17	19	29
265	1	1	1	0	1	0	0
278	1	0	1	1	0	0	1
269	0	0	0	0	1	0	0
299	0	1	1	0	1	1	0
307	0	1	0	1	0	0	1
316	0	0	0	0	1	0	0

We now solve (the matrix is transposed as we solve $A\underline{\mathbf{v}} = \underline{\mathbf{0}}$ and not $\underline{\mathbf{v}}A = \underline{\mathbf{0}}$):

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix} \cdot \underline{\mathbf{v}} = \underline{\mathbf{0}}$$

modulo 2. One solution is

$$\underline{\mathbf{v}} = (1, 1, 1, 0, 1, 0)$$

We thus take the 1st, 2nd, 3rd and the 4th x-value and get

$$\begin{aligned} x &= 265 \cdot 278 \cdot 296 \cdot 307 = 6694540240 \equiv 34757 \pmod{n} \\ y &= \sqrt{(265^2 - n) \cdot (278^2 - n) \cdot (296^2 - n) \cdot (307^2 - n)} \\ &= 2 \cdot 3^4 \cdot 13^2 \cdot 17 \cdot 29 = 13497354 \equiv 28052 \pmod{n} \end{aligned}$$

This yields the gcds:

$$\gcd(x - y, n) = 149, \quad \gcd(x + y, n) = 587$$

which give a factorization

x	2	3	13	17	19	29	$x^2 - n$ splits
261	X				X		
262		X		X			
263	X	X					
264							
265	X	X	X	X			$-2 \cdot 3 \cdot 13^2 \cdot 17$
266		X					
267	X						
268		X	X				
269	X	X					
270							
271	X	X			X		
272		X					
273	X					X	
274		X					
275	X	X					
276							
277	X	X					
278		X	X			X	$-3^3 \cdot 13 \cdot 29$
279	X			X			
280		X			X		
281	X	X	X				
282				X			
283	X	X					
284		X					
285	X						
286		X					
287	X	X					
288							
289	X	X					
290		X			X		
291	X		X				
292		X					
293	X	X					
294			X				
295	X	X					

x	2	3	13	17	19	29	$x^2 - n$ splits
296		X		X			$3^2 \cdot 17$
297	X						
298		X					
299	X	X		X	X		$2 \cdot 3 \cdot 17 \cdot 19$
300							
301	X	X					
302		X				X	
303	X						
304		X	X				
305	X	X					
306							
307	X	X	X			X	$2 \cdot 3^2 \cdot 13 \cdot 29$
308		X					
309	X				X		
310		X					
311	X	X					
312							
313	X	X		X			
314		X					
315	X						
316		X		X			$3^6 \cdot 17$
317	X	X	X				
318					X		
319	X	X					
320		X	X				
321	X						
322		X					
323	X	X					
324							
325	X	X					
326		X					
327	X						
328		X			X		
329	X	X					
330			X	X			